Elastic Distortion Calculations on a Special Piston Gauge (PG27) up to 28 MPa in Different Operational Modes

G. F. Molinar, P. C. Cresto, C. Ehrlich and
J. Houck

Abstract. The elastic distortions of a special hydraulic piston gauge (PG27) operating at up to 28 MPa in simple, re-entrant and controlled-clearance configurations were calculated by an iterative analytical technique. It was possible to determine the radial displacements of the piston and the cylinder at any pressure, and the pressure distribution in the clearance with different fluids, and to compute the pressure distortion coefficient of the piston-cylinder unit under conditions of use. Calculations show that the elastic distortion, the pressure distribution in the clearance and the pressure distortion coefficients are not fluid-dependent, at least for two of the most commonly used fluids. In the simple configuration, the radial distortion of the piston-cylinder unit at the maximum pressure is of the same magnitude as the undistorted radial clearance (0,527 μm). Agreement between the experimental and the calculated pressure distortion coefficients is about 1,4 % and corresponds to less than 0,6 part per million difference in effective area at 28 MPa. In the re-entrant configuration, the clearance between the piston and cylinder decreases to a minimum value of 0,282 μm at 28 MPa. Agreement between the experimental and the calculated pressure distortion coefficients is within 14,3 % and corresponds to 8,0 ppm difference in effective area at 28 MPa. In the controlled-clearance configuration, the distortions of the cylinder are, on average, equal to zero, as expected with controlled-clearance piston gauges, and the pressure distribution in the clearance is the most linear of the three operating modes. Agreement between the experimental and the calculated pressure distortion coefficients corresponds to less than 11,0 ppm difference in effective area at 14 MPa and 28 MPa.

1. Introduction

It is not infrequent in international intercomparisons to find differences of 20 % to 30 %, sometimes even higher, in the determination of the pressure distortion coefficient of a piston gauge. Sometimes these differences are found not only at very high pressures, but also at moderate pressures well below 100 MPa [1].

A special and unique piston gauge, denoted PG27, was designed and constructed by the National Institute of Standards and Technology (NIST) in the USA in such a way that the system can be operated in *simple, re-entrant* and *controlled-clearance* configurations while keeping the same piston-cylinder combination and without changing the position of the piston relative to the cylinder [2]. As this piston gauge can be operated up to 28 MPa, it can be used to compare, even at moderate pressures, the pressure distortion coefficients of the three configurations.

While PG27 was being characterized experimentally, the Istituto di Metrologia "G. Colonnetti" (IMGC), Italy, developed an analytical iterative method useful for computing the elastic distortions in the piston and cylinder for any piston-cylinder geometry and, consequently, the pressure distortion coefficient of the unit [3, 4].

This paper describes the PG27 piston gauge, a discussion of the calculated and experimental results of the piston and cylinder elastic distortions and their impact on the pressure distribution in the clearance for the three different configurations, and a comparison of the calculated and the experimentally obtained values of the pressure distortion coefficients for the three configurations.

G. F. Molinar and P. C. Cresto: Consiglio Nazionale delle Ricerche, Istituto di Metrologia "G. Colonnetti", Strada delle Cacce 73, I-10135 Torino, Italy.

C. Ehrlich and J. Houck: Thermophysics Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, USA.

2. Piston Gauge (PG27) Description

A schematic representation of PG27 providing details of the piston-cylinder region, complete with essential dimensional information for the calculations that are performed below, is presented in Figure 1. Other information pertaining to critical parameters of PG27 in its three configurational modes, and used here in the calculations, can be found in Table 1. Additional information on PG27 is given in [2].

The effective area A_{cc} of a controlled clearance piston gauge is typically expressed as [5]

$$A_{cc} = A_0\left(1 + b_p P\right)\left[1 + d\left(P_z - P_j\right)\right]\left[1 + \left(\alpha_p + \alpha_c\right)\left(T - T_r\right)\right], \quad (1)$$

where A_0 is the area of the piston at the reference temperature (T_r) for zero applied pressure, b_p is the pressure distortion coefficient of the piston, P is the pressure at the reference level of the piston, d is the jacket pressure coefficient, P_z is the zero-clearance jacket pressure, P_j is the operating jacket pressure, α_p and α_c are the respective linear thermal expansion coefficients of the piston and cylinder, and T is the temperature of the piston and cylinder. The experimentally determined pressure distortion coefficients (λ_c) for PG27 in the three configurations are obtained from the characterization of PG27 in the controlled-clearance configuration. To obtain λ_{cs} for the simple configuration, P_j is set equal to zero in (1). To
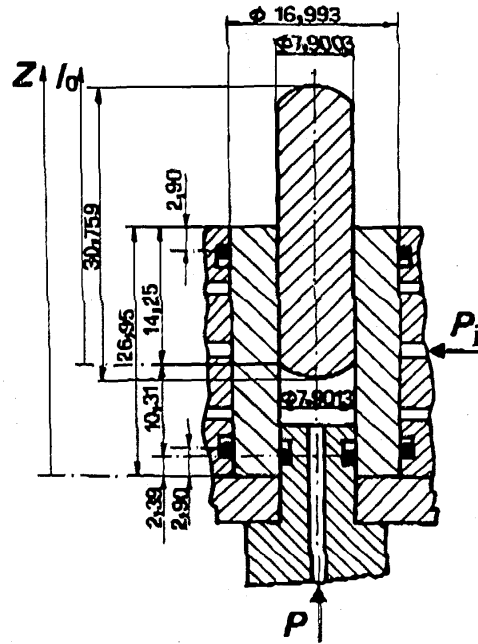

Figure 1. Schematic diagram of the PG27 piston gauge with the piston in its working position (undistorted radial clearance of 0.527 μm). Dimensions in mm are used to define the computer model for elastic distortion calculations in the three basic configuration modes: *simple*, $P_j = 0$; *re-entrant*, $P_j = P$; *controlled-clearance*, P_j values different from P.

Table 1. Basic parameters used to represent the main characteristics of the PG27 piston gauge in its three configuration modes.

Parameter	PG27, 28 MPa full scale
Outer cylinder diam./inner cylinder diam.	2,15
Radial clearance/μm	0,527
Radial clearance/piston radius	$0,133 \times 10^{-3}$
Piston-Cylinder engagement length/cylinder length	0,529
Piston-Cylinder engagement length/piston length	0,463 3
Maximum torque on upper retaining nut	Less than 2,5 N m
Pressure seal	By O-ring (Figure 1)
Pressure (gauge) on external cylinder	$P_j = 0$ simple $P_j = P$ re-entrant $P_j \neq P$ controlled-clearance
E_p/MPa; v_p	$6,3 \times 10^5$; 0,214
E_c/MPa; v_c	$6,3 \times 10^5$; 0,214
Material of piston	Tungsten carbide
Material of cylinder	Tungsten carbide
Fluid	Mineral oil
Maximum mass at full scale/kg	140
A_0 piston at 23 °C/mm²	49,021 39
A_0 piston uncertainty (1 σ)	± 6,5 ppm
λ_c/MPa⁻¹ experimental	Simple $1,15 \times 10^{-6}$ Re-entrant $-1,96 \times 10^{-6}$ Controlled-clearance $0,075 \times 10^{-6}$
$[(\alpha_p + \alpha_c) \pm \delta(\alpha_p + \alpha_c)]/(°C^{-1})$	$(9 \pm 1) \times 10^{-6}$
P uncertainty (1 σ) (simple mode)	± 10 ppm
Reference paper	[2]

obtain λ_{cr} for the re-entrant configuration, P_j is set equal to P. To obtain λ_{cc} for the controlled-clearance configuration, the relationship $P_j/\text{Pa} = 3 \times 10^6 + 0{,}36 \times P/\text{Pa}$ is used. In all three cases, making the above substitutions leaves (1) in the form $A_e = A_0(1 + b_1P + b_2P^2)$, where the temperature terms have been ignored. From these three quadratic equations, coefficients for the best-fit linear equations $A_e = A_0(1 + \lambda_e P)$ were obtained for each. These experimental values of λ_e, given in Table 1, were obtained in order to make comparisons with calculated linear distortion coefficients, as given in Section 4.

3. Calculations of Elastic Distortions

A computer model for the geometry of PG27 illustrated in Figure 1 was developed at the IMGC. For a number of pressures up to 28 MPa, the piston and cylinder radial distortions were calculated by general iterative methods previously described [3, 4] and used for the study of other piston gauges up to 500 MPa. The iterative process is started by computing the distortion of the piston-cylinder assembly assuming a constant radial clearance (0,527 µm) and a linear pressure distribution in the clearance along the entire piston-cylinder engagement length. The distortion values obtained are then entered into the program for the calculation of a new pressure distribution, which is entered again into the program for distortion calculations. The iterative procedure is repeated until the distortion and pressure distributions are mutually

consistent. In the present case, convergence was obtained with four to seven iterations.

Calculations were carried out for a mineral oil and a synthetic oil as the pressurizing fluids. In spite of the large differences in surface tension, density and dynamic viscosity of these two fluids, the differences in the calculated piston-cylinder radial distortions and pressure distributions were found to be insignificant.

4. Results

Figures 2 and 3 show the radial distortions of the piston and of the cylinder calculated at pressures of 14 MPa and 28 MPa, respectively, and for each of the three PG27 configurations (simple, re-entrant and controlled-clearance). In both figures the radial distortions of the piston and of the cylinder obtained by iterative calculations are superimposed on the undistorted piston-cylinder radial clearance of 0,527 µm. Figures 2 and 3 indicate that:

(a) in the simple and re-entrant cases, the cylinder undergoes the largest distortions;

(b) in the controlled-clearance case, the cylinder undergoes the smallest distortions which are, on the average, equal to zero, as expected;

(c) the piston always undergoes small radial compression (from 0,05 µm to 0,1 µm) over the engagement length;

(d) the largest radial clearance occurs in the simple configuration; a maximum radial clearance of 0,928 µm
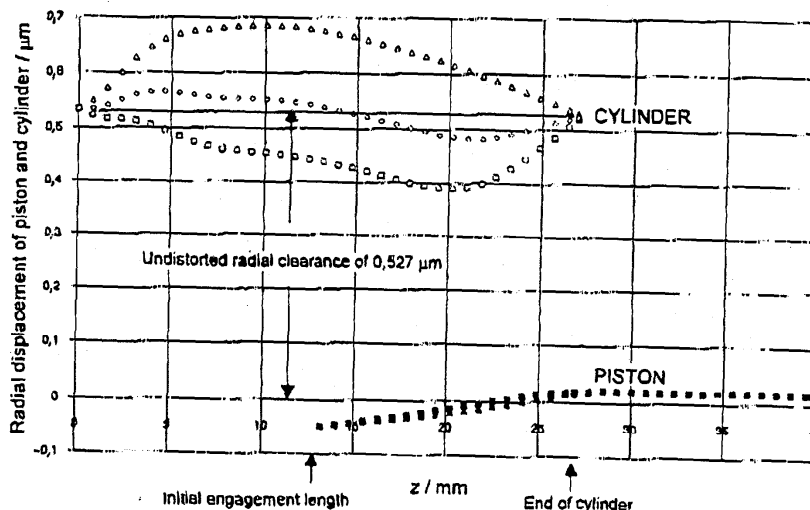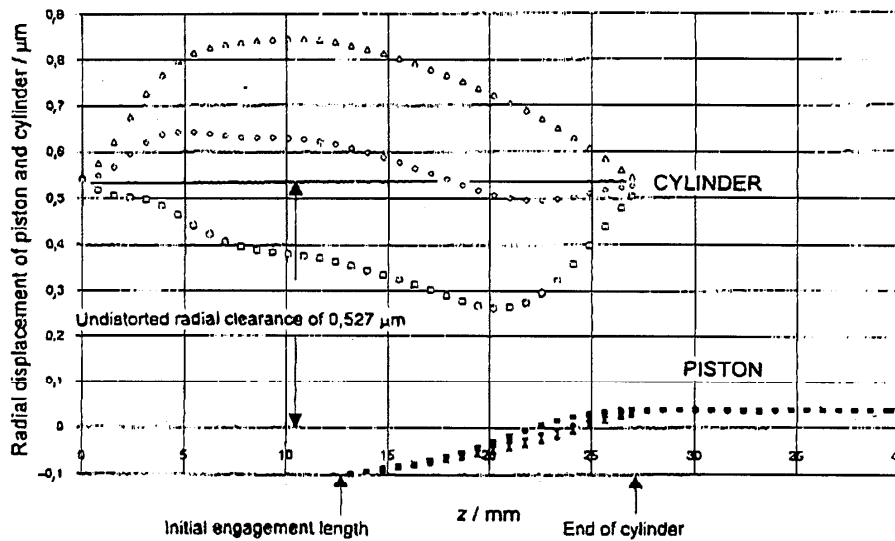
Figure 2. Calculated radial distortions of piston and cylinder of PG27 unit for 14 MPa for simple, re-entrant and controlled-clearance configurations.
Piston: ▲ simple; ■ re-entrant; ♦ controlled-clearance.
Cylinder: △ simple; □ re-entrant; ◊ controlled-clearance.

Figure 3. Calculated radial distortions of piston and cylinder of PG27 unit for 28 MPa for simple, re-entrant and controlled-clearance configurations.
Piston: ▲ simple; ■ re-entrant; ◆ controlled-clearance.
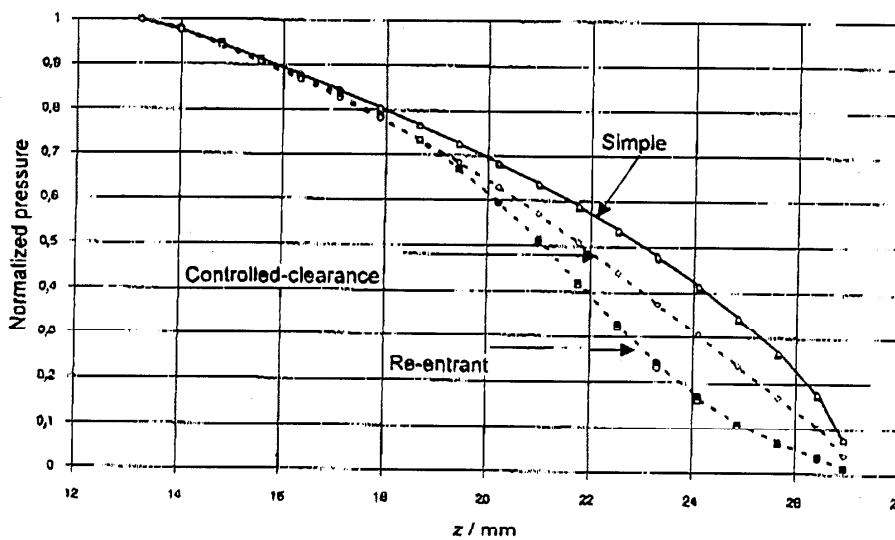Cylinder: △ simple; □ re-entrant; ◊ controlled-clearance.

Figure 4. Normalized pressure distribution in PG27 clearance versus piston-cylinder engagement length.
△: simple (mineral oil); ▼: simple (synthetic oil);
□: re-entrant (mineral oil); ■: re-entrant (synthetic oil); ◊:controlled-clearance.

takes place at 28 MPa; the radial distortions at 28 MPa are of the same magnitude as the undistorted initial radial clearance;

(e) the smallest radial clearance arises in the re-entrant configuration; a minimum radial clearance of 0,282 μm appears at 28 MPa.

The iterative method also allows the pressure distribution in the clearance to be calculated over the whole piston-cylinder engagement length.

The pressure distribution in the clearance ranges from the maximum pressure (at $\ell_0 = 0$ mm) to atmospheric pressure (at $\ell_0 = 14,25$ mm). The pressure distribution in the clearance of piston gauges often has a nonlinear behaviour [4, 6].

Even at the moderate pressures considered here, the pressure distribution in PG27 is not a linear function of engagement length and its behaviour depends on the piston-cylinder configuration.

As can be seen in Figure 4, the controlled-clearance configuration yields the most linear pressure distribution in the clearance. With the simple configuration, a markedly parabolic behaviour is calculated, while with the re-entrant configuration an S-shaped distribution is obtained. Pressure distributions of these types are frequently obtained when the piston-cylinder clearance is small; the same effects were previously demonstrated [4] in the case of piston-cylinder assemblies operating at pressures up to 500 MPa. From the radial distortions of the piston and cylinder and from the values of pressure distribution in the clearance, it is possible to compute the pressure distortion coefficient λ, for the three different configurations [4]. Such values are compared with the experimental values of Table 1.

For the *simple* configuration, the λ value obtained by calculation is $1,134 \cdot 10^{-6}$ MPa^{-1}, which differs by about 1,4 % (equivalent to 0,6 ppm* in effective area at 28 MPa) from the experimental value $(1,15 \cdot 10^{-6}$ MPa$^{-1})$. The calculated value proved to be fluid- and pressure-independent.

For the *re-entrant* configuration, the λ value obtained by calculation is $-2,24 \cdot 10^{-6}$ MPa^{-1}, which differs by about 14,3 % (equivalent to 8 ppm in effective area at 28 MPa) from the experimental value $(-1,96 \cdot 10^{-6}$ MPa^{-1} for the gauge NIST1R in [2]). The calculated value proved to be fluid- and pressure-independent. The difference of 8 ppm in the effective areas at 28 MPa is acceptable in view of the 3 σ random uncertainty of 6 ppm in the experimental effective area [2]. It must be considered also that in the re-entrant configuration the jacket pressure values are beyond the range of those used during the fall-rate measurements reported in Figure 2 of [2].

For the *controlled-clearance* configuration, the λ value obtained by calculation is $-0,279 \cdot 10^{-6}$ MPa^{-1}, while the experimental value was $+0,0754 \cdot 10^{-6}$ MPa^{-1}.

1 ppm = 1×10^{-6}.

This corresponds to a difference in effective area at 28 MPa of 10 ppm. The same calculations performed at 14 MPa showed a λ difference, with respect to the experimental value, equivalent to a difference of about 11 ppm in effective areas.

It must be noted that the experimental λ values mentioned above are, with all three configurations, intentionally referred to a linear behaviour of the effective area, as discussed in Section 2, in order to allow a convenient comparison.

For the controlled-clearance configuration, the same experimentally determined jacket pressure versus applied pressure relationship, as given in Section 2, was used for both the measurements and calculations.

5. Conclusions

Elastic distortion calculations for the PG27 piston gauge operating in liquid media up to 28 MPa were made by application of the IMGC analytical iterative procedure. From four to seven iterations were required before obtaining solution convergence.

Calculated piston and cylinder distortions, and the pressure distribution in the clearance, were proved to be independent of the two fluids used. The two fluids were selected from those commonly used in pressure metrology.

The calculated pressure distortion coefficients were proved to be pressure-independent for the three configurations.

The *simple* PG27 configuration gave very satisfactory calculated results; agreement between effective areas using the calculated and the experimental values of λ is within 1 ppm over the entire experimental pressure range. If the pressure distortion coefficient is calculated using a simplified theory [5] one obtains $\lambda = 1,12 \cdot 10^{-6}$ MPa^{-1}, which is in very good agreement with both experimental and calculated values. This provides further confidence that the simplified calculation of λ is valid for simple and regular geometries and moderate pressures when the elastic constants of the materials are accurately known.

The calculation of λ for the *re-entrant* configuration also compares satisfactorily with the experimental value. However, because of the small radial clearance (0,282 μm at 28 MPa), the reproducibility of the experimentally obtained effective area is not as good as that of the simple or of the controlled-clearance configurations [2].

The *controlled-clearance* configuration has the smallest cylinder distortion and the most linear pressure distribution in the clearance. At pressure values of 14 MPa and 28 MPa the calculations agree with the experimental results for the effective area to within 11 ppm. Further, the calculations predict the anticipated advantages of controlled-clearance methods.

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